

# Priority Queueing With Finite Buffer Size and Randomized Push-out Mechanism

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**Abstract.** The preemptive priority queueing with a finite buffer is considered. We introduce a randomized push-out buffer management mechanism which allows to control very efficiently the loss probability of priority packets. The packet loss probabilities for priority and non-priority traffic are calculated using the generating function approach. For the particular case of the standard non-randomized push-out scheme we obtain explicit analytic expressions. The theoretical results are illustrated by a numerical example.

**Keywords** — *priority queueing; transport protocols; finite buffer; traffic, randomized push-out*

## I. INTRODUCTION

The escalating role of telecommunication in the modern world demands deeper and more detailed understanding of the processes proceeding in computer networks. These processes make direct impact on a network overall performance as a whole.

Main purpose of development of computer networks is to increase their performance by means of new methods and algorithms of a traffic control. But for working out these methods it is necessary to represent, what is the network, what processes proceed in it and what influences its performance. To answer these questions it is necessary to develop network model.

Telematics networks can be considered as data flows from various network applications. In this article network connections are considered at transport layer. There are two classes of transport connections: the first class (TCP connections) – packets with the guaranteed delivery and concerning low throughput. The second class (UDP connections) are stream data with the minimum delay and high throughput.

Many applications simultaneously use both TCP and UDP connections. The packets control strongly influences quality of functioning of such applications. For these control purposes telematics devices such as routers, gateways and firewalls can be used. All such devices have limited memory buffer used for reception and transfer of packets. The state of computer networks (current level of their loading, a random variable of a delay and the maximum throughput) can essentially vary in the course of functioning, therefore the rules of priority service, at processing of the traffic from various applications, essentially influences quality of service.

Opportunities to increase efficiency of the packets management appear while using the priority service

system with finite buffer size and randomized push-out mechanism. Thus, the control parameter is the probability of push-out a priority packets. We will show principles of a choice of optimum push-out probability.

The paper is organized as follows. In Section 2 the mathematical model of the telematics device is shown. In Section 3 main equations and received results are shown. In Section 4 the future practical usage area of our priority mechanism is described. Section 5 is the conclusion and the overall results are discussed and potential future research is offered.

## II. MODEL OF NETWORK ENVIRONMENT

Here we consider the preemptive priority queueing system with two classes of packets. Class 1 packets have priority over class 2 packets. The packets of class 1 (2) arrive into the buffer according to the Poisson process with rate  $\lambda_1$  ( $\lambda_2$ ). The service time has the exponential distribution with the same rate  $\mu$  for each class. The service times are independent of the arrival processes. The buffer has a finite size  $k$  ( $1 < k < \infty$ ) and it is shared by both types of customers. The absolute priority in service is given packets of the first class. Unlike typical priority queueing considered system is supplied by the randomized push-out mechanism. If the buffer is full, a new coming customer of class 1 can push out of the buffer a customer of class 2 with the probability  $\alpha$ . Note that if  $\alpha = 1$  we retrieve the standard non-randomized push-out.

The scheme described priority queueing is resulted on Fig. 1. The priority queueing without the push-out mechanism ( $\alpha = 0$ ) and with the determined push-out mechanism ( $\alpha = 1$ ) were studied by G.P.Basharin. The concept of the randomized push-out mechanism with reference to network and telecommunication problems is offered in [1] where this mechanism was combined with relative priority, instead of absolute, as in our case.

The summarized entering stream represented on Fig. 1, will be the elementary with intensity  $\lambda = \lambda_1 + \lambda_2$ . If we'll trace only the general number of packets in system, then simplified one-data-flow model would be  $M/M/1/k$  type. The special modification of standard Kendel notation intended for priority queueing was proposed by G.P.Basharin. In the modified system the general structure of a label and sense of its separate positions remains, however in each position the vectorial

symbolics is used. There is an additional symbol  $f_i^j$ , where  $i$  specifies priority type (0 – without a priority, 1 – relative, 2 – absolute), and  $j$  specifies a type of the pushing out mechanism (0 – without pushing out, 2 – the determined pushing out).

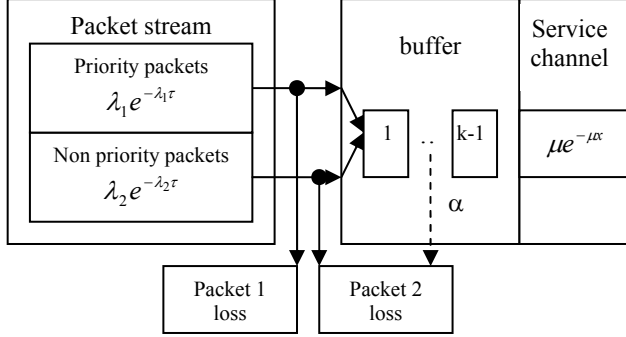


Figure 1. Priority queueing schema  $\bar{M}_2 / M / 1 / k / f_2^1$ .

The value  $j=1$  has not been involved in original system. In our opinion, it is expedient to use this label for the randomized push-out mechanism, as an intermediate between variants  $j=0$  and  $j=2$ . So the priority queueing represented on Fig. 1, is  $\bar{M}_2 / M / 1 / k / f_2^1$  type.

The history of research of one-channel two-data-flow priority systems includes already more than half a century, however, as far as we know, there is only one work [1] where the randomized push-out mechanism have been studied (in a combination with the relative priority for queueing  $\bar{M}_2 / M / 1 / k / f_1^1$  type). At the same time, for the typical models with the push-out mechanism ( $j=0$  and  $j=2$ ) the problem is solved basically.

Problems of research priority queueing, considered in some works originally have arisen in telecommunication with the analysis of real disciplines of scheduling in operating computers. Last years a similar sort of queueing model, and also their various generalisations are widely used at the theoretical analysis of real Internet systems.

The interesting problem is solved by A.Bondi [3]. In this work typical models with the general buffer  $\bar{M}_2 / M / 1 / k / f_2^0$  and separate buffers  $\bar{M}_2 / M / 1 / \bar{k} / f_2^0$  are compared to partially divided buffers at preservation of some general part. In work [4] the threshold push-out mechanism at which pushing out is authorised only at enough big length of not priority packets buffer is considered. Wagner and Krieger analyzed the M/M/1/K type non-preemptive priority queueing with the complete sharing buffer management scheme and with the class-dependent service rates. Cheng and Akyildiz considered the priority queueing with general service time distributions and a general service discipline function. They analyzed the push-out with threshold as the buffer management scheme. There are also some work on finite capacity priority queues [7,8].

As shown in [1], the probability pushing out mechanism is more convenient and effective in comparison with other mathematical models of pushing out considered in the literature. It adequately describes real processes of the network traffic and is thus simple enough from the mathematical point of view.

### III. MAIN EQUATIONS

The state graph of system  $\bar{M}_2 / M / 1 / k / f_2^1$  is presented on Fig. 2.

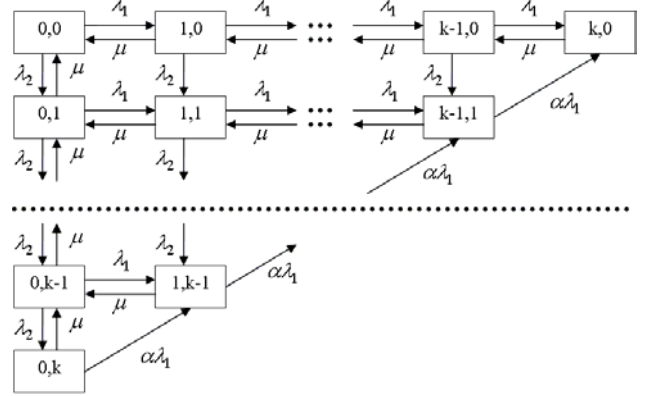


Figure 2. The state graph of  $\bar{M}_2 / M / 1 / k / f_2^1$  type system.

Making by usual Kolmogorov's rules [2] set of equations with the help of state graph we will receive:

$$\begin{aligned}
 & -[\lambda_1(1 - \delta_{j,k-i}) + \alpha\lambda_1(1 - \delta_{j,k})]\delta_{j,k-i} + \lambda_2(1 - \delta_{j,k-i}) + \\
 & + \mu(1 - \delta_{i,0}\delta_{j,0})p_{ij} + \mu p_{i+1,j} + \mu\delta_{i,0}p_{i,j+1} + \lambda_2 p_{i,j-1} + \\
 & + \lambda_1 p_{i-1,j} + \alpha\lambda_1 \delta_{j,k-i} p_{i-1,j+1} = 0, \quad (i = \overline{0, k}; j = \overline{0, k-i}),
 \end{aligned} \quad (1)$$

where  $\delta_{i,j}$  is the delta-symbol. The system should dare together with a normalization condition

$$\sum_{i=0}^k \sum_{j=0}^{k-i} p_{ij} = 1.$$

At real  $k$  (big enough) this system is ill-conditioned, and its numerical solution leads to the big computing errors. Many authors, in particular, G.P.Basharin bypass this difficulty, applying a method of recurrent relations. In this method the problem is reduced to a solution of some auxiliary system of the linear algebraic equations of an  $\frac{1}{2}k(k+1)$  order with a triangular matrix.

In the present paper we use the method of generating functions [1] in its classical variant offered by H.White, L.S.Christie and F.F.Stephan with reference to  $\bar{M}_2 / M / 1 / f_2$  type systems [5,6].

Solving (1) system we receive some auxiliary variables

$$\begin{aligned}
 p_i &= p_{k-i}, \quad (i = \overline{0, k}), \\
 q_{k-j} &= (1 - \alpha) \sum_{i=1}^j p_i \rho_1^{i-j} + q_k \rho_1^{-j}, \quad (j = \overline{1, k}), \\
 r_n &= \frac{(1 - \rho) \rho^n}{(1 - \rho^{k+1})}, \quad (n = \overline{0, k}).
 \end{aligned}$$

When using them we can receive loss probability for priority ( $p_{loss}^{(1)}$ ) and non-priority ( $p_{loss}^{(2)}$ ) packets:

$$r_n = \frac{(1-\rho)\rho^n}{(1-\rho^{k+1})},$$

$$p_{loss}^{(2)} = r_k + \alpha \frac{\rho_1}{\rho_2} \sum_{i=1}^k p_i + \frac{\rho_1}{\rho_2} p_k$$

We also receive waiting probability and mean number of load service channels:

$$p_{wait}^{(1)} = 1 - q_0 - q_k, p_{wait}^{(2)} = 1 - r_0 - r_k.$$

$$\bar{n}_{load}^{(1)} = 1 - q_0, \bar{n}_{load}^{(2)} = q_0 - r_0.$$

By these formulas we received some graphs for different rate of input streams:

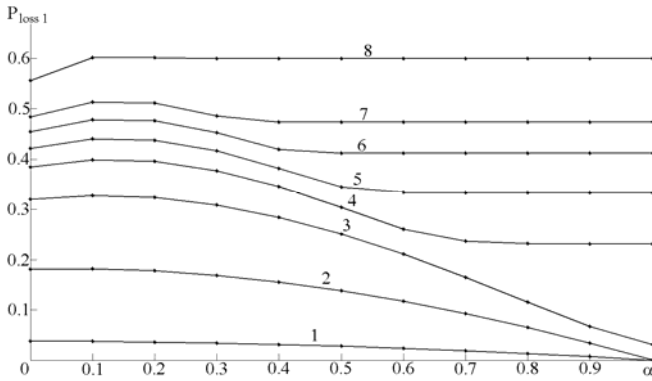


Figure 3. Loss probability for priority ( $p_{loss}^{(1)}$ ) packets for strongly loaded transport virtual channel with  $\rho_2 = 1,5$  and different values  $\rho_1$ :

1 -  $\rho_1 = 0,1$ ; 2 -  $\rho_1 = 0,5$ ; 3 -  $\rho_1 = 1,0$ ; 4 -  $\rho_1 = 1,3$ ; 5 -  $\rho_1 = 1,5$ ; 6 -  $\rho_1 = 1,7$ ; 7 -  $\rho_1 = 1,9$ ; 8 -  $\rho_1 = 2,5$ . The same legend is used by all Figures.

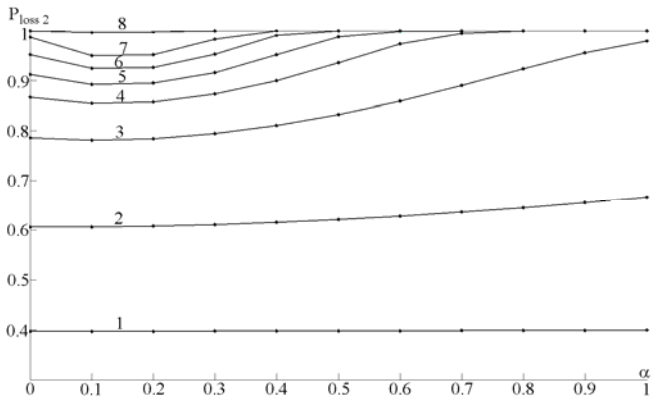


Figure 4. Loss probability for priority ( $p_{loss}^{(2)}$ ) packets for strongly loaded transport virtual channel. The legend is described on Fig.3.

From Fig. 3 and 4 we can see, that choosing parameter  $\alpha$ , we can influence on  $P_{loss}^{(i)}$  in very wide limits. For some values  $\rho_1$  the range of changes of  $P_{loss}^{(i)}$  will be 3-30 % at  $\lambda_1 + \lambda_2 \gg \mu$ . On schedules the extremum is obviously visible at values  $\alpha = 0,1 - 0,2$ , it means that increasing probability of replacement of not priority packets, we thus reduce probability of their loss in the strongly loaded networks. It is possible to explain it by

that in the absence of replacement mechanism ( $\alpha = 0$ ) and at  $\alpha > 0$  various mechanisms work.

The loss probability defines throughput of system. There are an absolute throughput of i-packet type  $\lambda_i(1 - P_{loss}^{(i)})$ , relative throughput of this type

$$\mathfrak{a}_i = 1 - P_{loss}^{(i)}, (i = \overline{1,2})$$

and mutual throughput of i-packets type, concerning j-packets type

$$\mathfrak{a}_{ij} = \mathfrak{a}_i / \mathfrak{a}_j (= (1 - P_{loss}^{(i)}) / (1 - P_{loss}^{(j)})).$$

The relative time that the priority packet spend in queueing by Little's Formula

$$\theta_i = \frac{\bar{s}_i}{\bar{\tau}_i} = \frac{\bar{n}_{load}^{(i)}}{(1 - P_{loss}^{(i)})} + \rho_i, \bar{\tau}_i = \frac{1}{\lambda_i}, (i = \overline{1,2}).$$

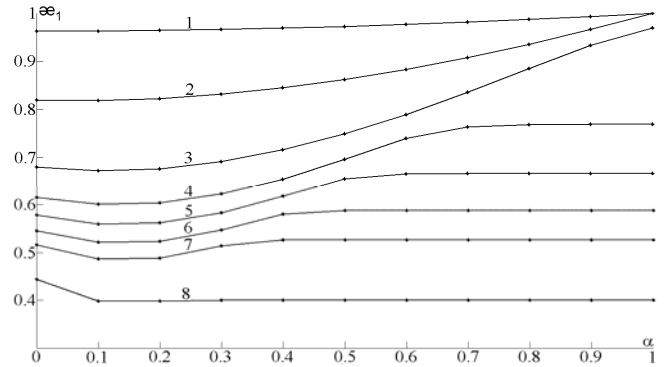


Figure 5. Relative throughput of priority packets

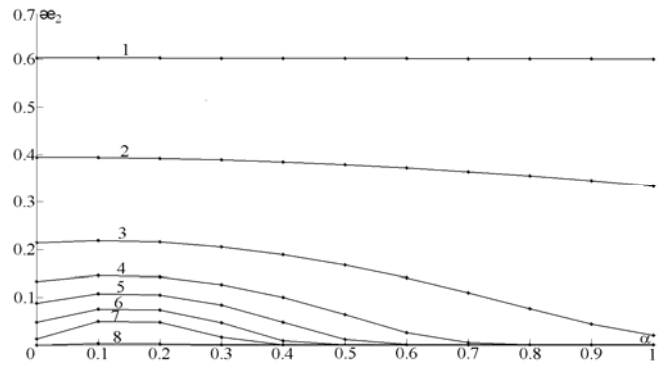


Figure 6. Relative throughput of non-priority packets

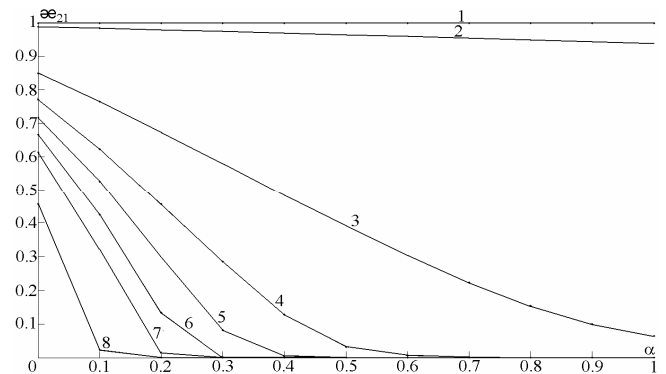


Figure 7. Throughput2/Throughput1, low load ( $\rho_2 = 0,5$ )

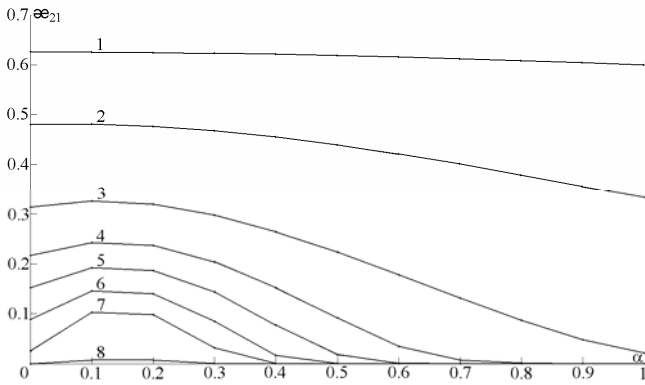


Figure 8. Throughput2/Throughput1, heavy load ( $\rho_2 = 1,5$ )

Nonlinear character of the received graphs can be explained by the overflow in virtual transport channels when besides control data in the channel there is an additional information, for example a video data stream.

For the comparison with data of job [1] we've set the same capacity of the buffer size (30 packets) that corresponds  $k = 31$ .

We've study dependence of  $P_{loss}^{(i)}$  replacement parameter in  $\alpha$  a mode of moderate loading at prevalence of not priority requirements ( $\rho_1 = 0.2, \rho_2 = 0.9$ ). Results for our system and system [1] are presented on Fig 9.

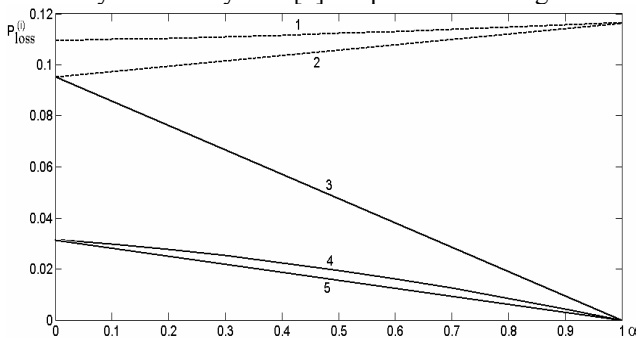


Figure 9. The comparison of loss probability for priority ( $P_{loss}^{(1)}$ ) and non-priority ( $P_{loss}^{(2)}$ ) packets between systems with absolute and relative priority.

The replacement parameter  $\alpha$  allows to operate effectively loss probability of priority packets in very wide limits. So an increase of  $\alpha$  from 0 to 1 value decreases  $P_{loss}^{(i)}$  more than by  $1,836 \cdot 10^{20}$  times. Thus the loss probability of not priority packet increases only by 6,4 %.

We have to mention, that introduction of an absolute priority influences throughput of a computer network much more poorly, than  $\alpha$  change. So, by changing the priority from relative to absolute value decreases  $P_{loss}^{(1)}$  only by 2-3 times.

It is important to notice, that dependence received  $P_{loss}^{(i)}(\alpha)$  numerical by, it is very close to linear for both types of packets. In practice of engineering calculations at the weak priority traffic linear approximation is

comprehensible  $\rho_1/\rho_2$ . In case of prevalence of priority packets the picture cardinally varies. On fig. 10 it is shown the dependence of  $P_{loss}^{(i)}$  from  $\alpha$ .

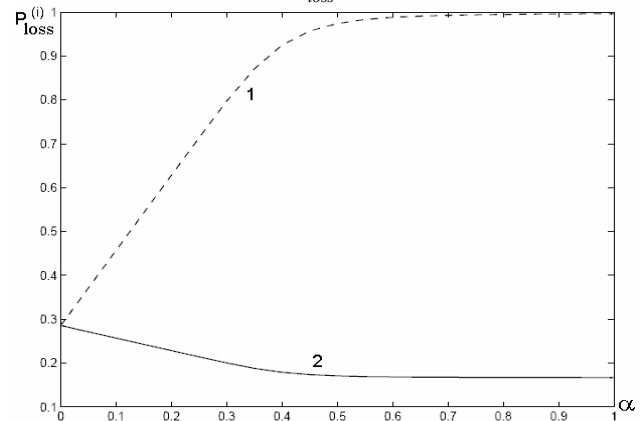


Figure 10.  $P_{loss}^{(i)}(\alpha)$  at  $\rho_1 = 1,2, \rho_2 = 0,2$ : 1 -  $i=2$ , an absolute priority; 2 -  $i=1$ , an absolute priority.

When  $\rho_1 > \rho_2$   $P_{loss}^{(i)}(\alpha)$  differs from linear for both types of packets and comes to the nearly constant level. We will notice, that curves on Fig. 10 practically repeat the similar curves in [1] constructed for a case of a relative priority. Thus, for strongly loaded network the priority type is much less important, than presence of the push-out mechanism and a value of  $\alpha$  parameter. The push-out mechanism allows to operate the network traffic and when the priority mechanism ceases to operate.

Fig. 11 shows the relative time that the priority packet spend in queueing. When the push-out probability of non-priority packet grow, a share and the relative time that priority packets have spend in queueing grow too.

It's interesting to study the intermediate conditions of network load in which the linear law of the losses has already been broken, but the saturation zone has not been generated yet. Numerical experiment has been made to detect conditions in which  $\rho_1$  varied over a wide range from 0,1 to 2,5, and  $\rho_2 = 1,5$ . The research objective was an analysis of network interactions at magnification of the priority traffic against constant and intensive stream of not priority packets. Received graphs are shown on Fig. 3,4,5,6,7,8,11.

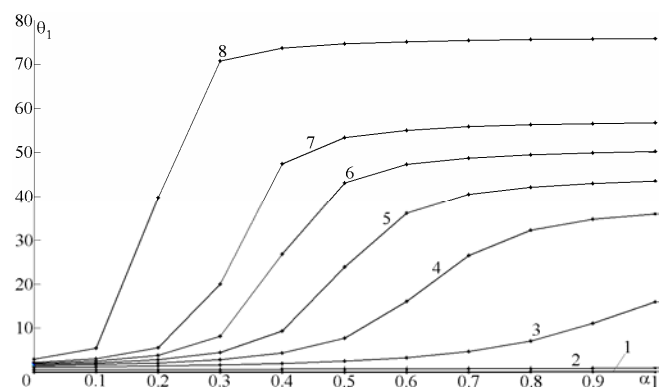


Figure 11. The time that priority packet spend in queueing

We noticed, that the complicated nonmonotone associations of network parameters on parameter  $\alpha$  take place only when the value of intensity of a background traffic of not priority packets  $\rho_2$ . We also researched the case of to moderate loading  $\rho_2 = 0,5$ , with the same values of  $\rho_1$ . On the graphs similar to graphs fig. 3,4,5,6,7,8,11, any extremum are absent, and all curves are monotone.

Our results show, that the described complicated nonlinear effects of network interaction are existing only in hardly loaded networks that is agree with the data of natural observations. Models of priority queueing with the probability push-out mechanism allow to model successfully these effects and to explain them.

#### IV. PRACTICAL USAGE OF PROPOSED MODEL

Good example of opportunity to use such mechanism is the problem of controlling removed robotic object which functioning outcomes, along with telemetry data and a video stream, are transmitted with the usage of global networks. In this case control commands are transmitted by TCP, and outcomes of observation represent a video stream data and are transmitted by UDP. A mean values of bandwidths of our robotic object: bandwidth of TCP channel (control and telemetry packets)  $\sim 100\text{Kb/s}$ , bandwidth of UDP video stream  $\sim 1,2\text{Mb/s}$ .

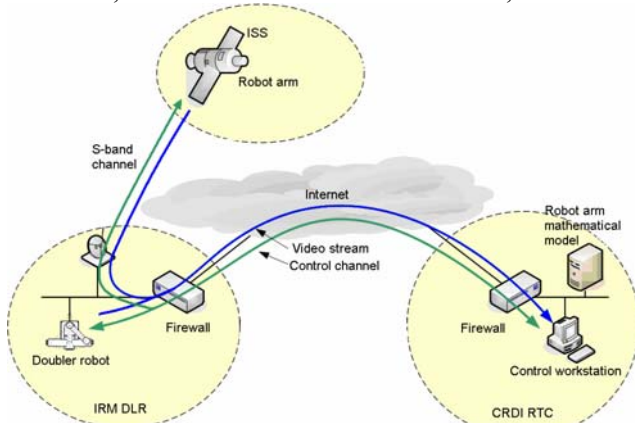


Figure 12. The scheme of space experiment "Contour"

In a considered example (ROKVISS mission [9]), the choice of a priority of service and loss-probability of a priority packet  $\alpha$ , allows to balance such indicators of functioning of a network, as loss-probability of control packets  $p_{loss}^{(1)}$  and quality of data of video observation for various conditions of a network environment. The parameter  $\alpha$  can vary for delay minimization in a contour of a feedback of a control system.

The given problem is important for interactive control of remote real-time dynamic objects, in a case when the complex computer network is the component of a feedback control contour, therefore minimization of losses and feedback delays, is the important parameter characterizing an effectiveness of control system.

#### V. CONCLUSION.

1. In the paper is stated the method of calculation the performances of a two data-flow computer network. TCP packets have the absolute service and queueing priority over UDP background traffic. The computer network is modelled by means of queueing with the finite buffer size and the probability push-out mechanism in a combination with an absolute priority. The offered effective computing algorithm allows network engineers and designers to estimate possible variants of network traffic load.

2. It was shown that in overload networks, then  $\lambda = \lambda_1 + \lambda_2 \gg \mu$ , the dependence of loss priority from variable  $\alpha$  is not linear and even not monotonous function. In that case the network with an absolute priority gives a maximum of advantages to the most important types of packets. Thus in comparison with a relative priority it is possible to reduce probability of displacement  $\alpha$  that makes our algorithm of control more flexible.

3. It was described the class of applications where our model is worth using and described the real practical application of presented model.

4. Proposed approach sheds new light on the problem of traffic control and could lead to more efficient behavior of routers and other network appliances.

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